1 $A = \{-1, 1\}$, $B = \{0, 2, 3\}$

i) $A \times B = \{-1, 1\} \times \{0, 2, 3\}$

$= \{(-1,0), (-1,2), (-1,3), (1,0), (1,2), (1,3)\}$

ii) Number of relations from A to B $= 2^6 = 64$

iii) $B = \{0, 2, 3\}$

Subsets of B

$\{0\}, \{2\}, \{3\}, \{0,2\}, \{0,3\}, \{2,3\}, \{0,2,3\}, \phi$

2 i) c) $\left[\pi, \frac{3\pi}{2}\right]$

ii) $\cos x = -\frac{4}{5}$

iii $\frac{5\pi}{3} = \left(\frac{5\pi}{3} \times \frac{180}{\pi}\right)^{\circ}$

$= 300^{\circ}$

3. $$\frac{4-3x}{2} \geq \frac{1-x}{4} - 2$$

$$4-3x \geq \frac{1-x}{2} - 4$$

$$4-3x \geq \frac{1-x-8}{2}$$

$$2(4-3x) \geq -7-x$$

$$8-6x \geq -7-x$$

$$-6x+x \geq -7-8$$

$$-5x \geq -15$$

$$x \leq 3$$

Solution is $(-\infty, 3]$

4. i) $^{4}C_{1} \times {}^{48}C_{4} = 4 \times 194580 = 778320$ ways.

ii) $^{n}C_{4} = {}^{n}C_{6} \Rightarrow n = 10$

$$^{n}C_{8} = {}^{10}C_{8} = \frac{10!}{8!\,2!} = \frac{10 \times 9}{1 \times 2} = 45$$

5. i) $2n+1$ terms

ii) $\left(x+\frac{3}{x}\right)^4 = {}^4C_0\, x^4 + {}^4C_1\, x^3 \cdot \left(\frac{3}{x}\right) + {}^4C_2\, x^2 \left(\frac{3}{x}\right)^2 + {}^4C_3\, x \left(\frac{3}{x}\right)^3 + {}^4C_4 \left(\frac{3}{x}\right)^4$

$= 1 \times x^4 + 4 \cdot x^3 \cdot \frac{3}{x} + 6 \cdot x^2 \cdot \frac{9}{x^2} + 4 \cdot x \cdot \frac{27}{x^3} + 1 \cdot \frac{81}{x^4}$

$= x^4 + 12x^2 + 54 + \frac{108}{x^2} + \frac{81}{x^4}$

6. i) $\frac{x}{-4} + \frac{y}{3} = 1$

$\frac{3x-4y}{-12} = 1, \quad 3x - 4y = -12$

$3x - 4y + 12 = 0$

ii) $\frac{|12|}{\sqrt{3^2+(-4)^2}} = \frac{12}{5}$

7. $r = 5$

centre lie on x axis $\rightarrow (h, 0)$

Equation of circle is

$$(x-h)^2 + y^2 = 25 \quad \text{—①}$$

① passing through $(2,3)$

$$(2-h)^2 + 3^2 = 25$$

$$(2-h)^2 = 16$$

$$2-h = \pm 4$$

$$h = -2, \; h = 6$$

when $h = -2$, equation of circle is

$$(x+2)^2 + y^2 = 25$$

$$x^2 + y^2 + 4x - 21 = 0$$

when $h = 6$, equation of circle is

$$(x-6)^2 + y^2 = 25$$

$$x^2 + y^2 - 12x + 11 = 0$$

8. i) $\lim_{x\to 0} \frac{\sin x}{x} = 1$

ii) $\lim_{x\to 0} \frac{1-\cos x}{\sin x} = \lim_{x\to 0} \frac{\frac{1-\cos x}{x}}{\frac{\sin x}{x}}$

$$= \frac{\lim_{x\to 0} \frac{1-\cos x}{x}}{\lim_{x\to 0} \frac{\sin x}{x}} = \frac{0}{1} = 0$$

9. i) $\{x : x \in R, -1 < x \leq 3\}$

ii) $A \cap A' = \phi$

iii) $U = \{1,2,3,4,5,6\}$, $A = \{2,3\}$, $B = \{3,4,5\}$

$A \cup B = \{2,3,4,5\}$

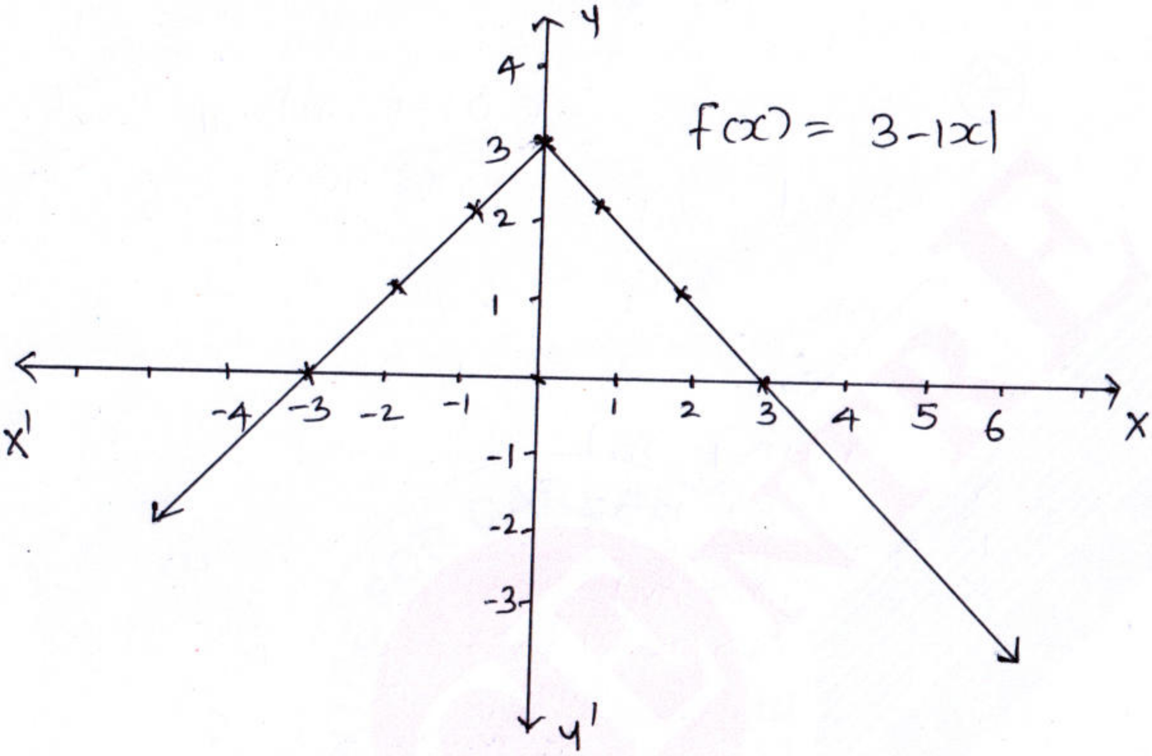
$B - A = \{4,5\}$

$A \cap B = \{3\}$

$(A \cup B)' = \{1,6\}$

10. i)

x	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-1	0	1	2	3	2	1	0	-1	-2	-3



ii) Range of $f(x) = (-\infty, 3]$

iii) $f(x) = \sqrt{4 - x^2}$

Domain of $f(x) = [-2, 2]$

11. i) $\frac{5+\sqrt{2}i}{1-\sqrt{2}i} = \frac{(5+\sqrt{2}i)(1+\sqrt{2}i)}{(1-\sqrt{2}i)(1+\sqrt{2}i)}$

$= \frac{5+5\sqrt{2}i+\sqrt{2}i-2}{1^2-(\sqrt{2}i)^2}$

$= \frac{3+6\sqrt{2}i}{3}$

$z = 1+2\sqrt{2}i$

ii) $z = 4-3i$

$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4+3i}{4^2+(-3)^2} = \frac{4+3i}{25}$

$z^{-1} = \frac{4}{25} + \frac{3}{25}i$

12. i) INDEPENDENCE

$\frac{12!}{3!\times 4!\times 2!} = 1663200$

$N \to 3$
$E \to 4$
$D \to 2$

ii $\boxed{\text{IEEEE} \mid \text{NDPNDNC}}$

$$\frac{5!}{4!} \times \frac{8!}{3! \times 2!} = 16800$$

13 i) Slope of l_1, $m_1 = \tan 30^\circ$

$$= \frac{1}{\sqrt{3}}$$

ii) $(0,0)$, $m = \frac{1}{\sqrt{3}}$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{1}{\sqrt{3}}(x - 0)$$

$$y = \frac{x}{\sqrt{3}}$$

$$\sqrt{3}y = x$$

$$x - \sqrt{3}y = 0$$

iii Slope of l_2, $m_2 = \frac{-1}{m_1} = \frac{-1}{1/\sqrt{3}} = -\sqrt{3}$

$(4,0)$ $m = -\sqrt{3}$

$$y - 0 = -\sqrt{3}(x - 4)$$

$$y = -\sqrt{3}x + 4\sqrt{3}$$

$$\sqrt{3}x + y - 4\sqrt{3} = 0$$

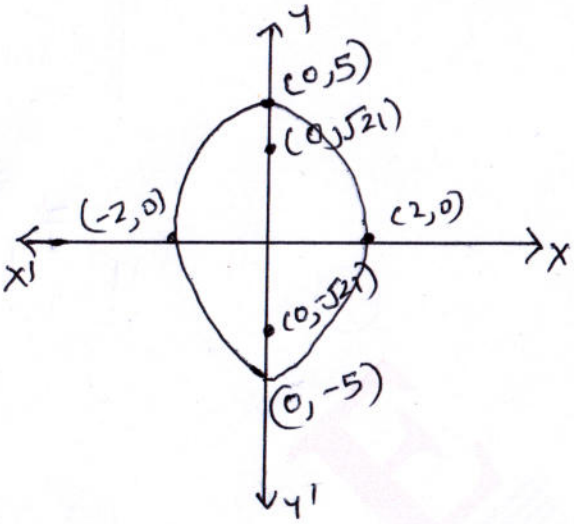
14. $\dfrac{x^2}{4} + \dfrac{y^2}{25} = 1$

$c^2 = a^2 - b^2$

$= 25 - 4$

$= 21$

$c = \sqrt{21}$



foci : $(0, \pm\sqrt{21})$

vertices : $(0, \pm 5)$

Eccentricity : $e = c/a = \dfrac{\sqrt{21}}{5}$

Length of latus rectum: $\dfrac{2b^2}{a} = 2 \times \dfrac{4}{5} = \dfrac{8}{5}$

15 i) 6^{th} octant

B) $(-3, 1, -2)$

ii) $A(-2,3,5)$, $B(1,2,3)$, $C(7,0,-1)$

$AB = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2} = \sqrt{14}$

$BC = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} = \sqrt{56} = 2\sqrt{14}$

$AC = \sqrt{(7--2)^2 + (0-3)^2 + (-1-5)^2} = \sqrt{126} = 3\sqrt{14}$

$AB+BC=\sqrt{14}+2\sqrt{14}$

$=3\sqrt{14}$

$AB+BC=AC$

Hence A, B, C are collinear

16. i) $S=\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$A=\{TTT\}$

$B=\{HTT, THT, TTH\}$

$C=\{HHT, HTH, THH, HHH\}$

ii) $A\cap B=\phi$, $B\cap C=\phi$, $A\cap C=\phi$

$\therefore$ A, B and C are mutually exclusive events.

iii) D: Exactly two tails

$D=\{HTT, THT, TTH\}$

$P(D)=\frac{3}{8}$

17 i) $\sin\frac{\pi}{6}=\frac{1}{2}$, $\sec\frac{\pi}{3}=2$, $\cot\frac{\pi}{4}=1$

$\sin\frac{5\pi}{6}=\sin(\pi-\frac{\pi}{6})=\sin\frac{\pi}{6}=\frac{1}{2}$

17. $3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4} =$

$3\times\frac{1}{2}\times 2 - 4\times\frac{1}{2}\times 1$

$= 3-2 = 1$

ii) $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} =$

$\dfrac{\cos 4x + \cos 2x + \cos 3x}{\sin 4x + \sin 2x + \sin 3x}$

$= \dfrac{2\cos 3x\cos x + \cos 3x}{2\sin 3x\cos x + \sin 3x}$

$= \dfrac{\cos 3x(2\cos x+1)}{\sin 3x(2\cos x+1)}$

$= \cot 3x$

18 i) $a_4 = (a_2)^2$, $a = -3$

$ar^3 = (ar)^2$

$ar^3 = a^2 r^2$

$r = a = -3$

7^{th} term, $a_7 = ar^6 = (-3)\cdot(-3)^6$

$= (-3)^7$

$= -2187$

ii $3, \frac{3}{2}, \frac{3}{4}, \ldots$

$a = 3 \quad r = \frac{1}{2}$

$$\frac{a(1-r^n)}{1-r} = \frac{3069}{512}$$

$$\frac{3\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}} = \frac{3069}{512}$$

$$3 \times 2\left(1-\left(\frac{1}{2}\right)^n\right) = \frac{3069}{512}$$

$$1-\left(\frac{1}{2}\right)^n = \frac{3069}{3 \times 2 \times 512}$$

$$1-\left(\frac{1}{2}\right)^n = \frac{1023}{1024}$$

$$1-\frac{1023}{1024} = \left(\frac{1}{2}\right)^n$$

$$\frac{1024-1023}{1024} = \left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{2}\right)^n = \frac{1}{1024}$$

$$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{10}$$

$$n = 10$$

19 i $f(x) = \tan x$

$$f(x+h) = \tan(x+h)$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\tan(x+h)-\tan x}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sin(x+h)\cos x - \cos(x+h)\sin x}{\cos(x+h)\cos x}}{h}$$

$$= \lim_{h \to 0} \frac{\sin((x+h)-x)}{\cos(x+h)\cos x} \times \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\sin h}{h\cos(x+h)\cos x}$$

$$= \lim_{h \to 0} \frac{\sin h}{h} \lim_{h \to 0} \frac{1}{\cos(x+h)\cos x}$$

$$f'(x) = 1 \times \frac{1}{\cos x \cdot \cos x} = \frac{1}{\cos^2 x} = \sec^2 x$$

ii) $f(x) = \dfrac{x+\cos x}{\tan x}$

$$f'(x) = \frac{\tan x \frac{d}{dx}(x+\cos x) - (x+\cos x)\frac{d}{dx}\tan x}{\tan^2 x}$$

$$= \frac{\tan x(1-\sin x) - (x+\cos x)\sec^2 x}{\tan^2 x}$$

20.

Class	f_i	x_i	$x_i f_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
10-20	2	15	30	-30	900	1800
20-30	3	25	75	-20	400	1200
30-40	8	35	280	-10	100	800
40-50	14	45	630	0	0	0
50-60	8	55	440	10	100	800
60-70	3	65	195	20	400	1200
70-80	2	75	150	30	900	1800
	$\Sigma f_i = N = 40$		$\Sigma x_i f_i = 1800$			$\Sigma f_i(x_i - \bar{x})^2 = 7600$

$$\text{Mean, } \bar{x} = \frac{1}{N} \Sigma x_i f_i = \frac{1800}{40} = 45$$

$$\text{Variance, } \sigma^2 = \frac{1}{N} \Sigma f_i (x_i - \bar{x})^2$$

$$= \frac{7600}{40} = 190$$

Standard Deviation,

$$SD, \sigma = \sqrt{190}$$

$$= 13.78$$