1 $A=\begin{bmatrix}3 & 2 & 1\\ 0 & 4 & -1\\ -2 & 1 & 3\end{bmatrix}$, $A^T=\begin{bmatrix}3 & 0 & -2\\ 2 & 4 & 1\\ 1 & -1 & 3\end{bmatrix}$

$$A+A^T=\begin{bmatrix}6 & 2 & -1\\ 2 & 8 & 0\\ -1 & 0 & 6\end{bmatrix}$$

$$P=\frac{1}{2}(A+A^T)=\begin{bmatrix}3 & 1 & -1/2\\ 1 & 4 & 0\\ -1/2 & 0 & 3\end{bmatrix}$$

$$A-A^T=\begin{bmatrix}0 & 2 & 3\\ -2 & 0 & -2\\ -3 & 2 & 0\end{bmatrix}$$

$$Q=\frac{1}{2}(A-A^T)=\begin{bmatrix}0 & 1 & 3/2\\ -1 & 0 & -1\\ -3/2 & 1 & 0\end{bmatrix}$$

$$P+Q=\begin{bmatrix}3 & 1 & -1/2\\ 1 & 4 & 0\\ -1/2 & 0 & 3\end{bmatrix}+\begin{bmatrix}0 & 1 & 3/2\\ -1 & 0 & -1\\ -3/2 & 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}3 & 2 & 1\\ 0 & 4 & -1\\ -2 & 1 & 3\end{bmatrix}$$

$$=A$$

2 - i) $\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right)$

$= \underline{\underline{-\frac{\pi}{6}}}$, $-\pi/6 \in \left[-\pi/2, \pi/2\right]$

ii) $\cos^{-1}\left(\cos 7\pi/6\right)$

$\cos\left(\frac{7\pi}{6}\right) = \cos\left(2\pi - \frac{5\pi}{6}\right)$

$= \cos\frac{5\pi}{6}$

$\cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \underline{\underline{\frac{5\pi}{6}}}$, $\frac{5\pi}{6} \in [0, \pi]$

3 - $f(x) = \begin{cases} x^2 - 1, & x < 2 \\ 4, & x = 2 \\ 2x - 1, & x > 2 \end{cases}$

when $x<2$, $f(x) = x^2 - 1$, which is continuous (polynomial function)

$x>2$, $f(x) = 2x-1$, which is continuous

at $\underline{x=2}$

$f(2) = 4$

L.H.L $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2-1$

$= 2^2-1 = 3$

R.H.L $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 2x-1$

$= 2\times 2-1 = 3$

L.H.L = R.H.L $\neq f(2)$

$\therefore$ f is not continuous at $x=2$

f is not a continuous function.

4. Graph of $f'(x)$ is given

i) $f(x)$ is increasing in $(-\infty,1)\cup(3,\infty)$

[Because in the interval $(-\infty,1)\cup(3,\infty)$ graph of $f'(x)$ is above x axis ie $f'(x)>0$]

$f(x)$ is decreasing in $(1,3)$

[Graph of $f'(x)$ is below x axis ie $f'(x)<0$]

ii) $x=1$ is a point of local maxima

$x=3$ is a point of local minima

5. $\displaystyle\int \frac{x-1}{x^2-4x-5}\,dx$

$$\frac{x-1}{x^2-4x-5} = \frac{x-1}{(x-5)(x+1)}$$

$$\frac{x-1}{(x-5)(x+1)} = \frac{A}{x-5} + \frac{B}{x+1}$$

$$= \frac{A(x+1) + B(x-5)}{(x-5)(x+1)}$$

$$x-1 = A(x+1) + B(x-5)$$

Put $x=5$,

$$4 = 6A \Rightarrow A = \frac{4}{6} = \frac{2}{3}$$

$x=-1$,

$$-2 = -6B \Rightarrow B = \frac{2}{6} = \frac{1}{3}$$

$$\int \frac{x-1}{(x-5)(x+1)}dx = \int \frac{\frac{2}{3}}{x-5}dx + \int \frac{\frac{1}{3}}{x+1}dx$$

$$= \frac{2}{3}\log|x-5| + \frac{1}{3}\log|x+1| + c$$

6. i) $\vec{AB} = (4-1)\hat{i} + (5-5)\hat{j} + (7-3)\hat{k}$

$= 3\hat{i} + 4\hat{k}$

ii) $|\vec{AB}| = \sqrt{3^2+4^2} = 5$

Unit vector along $\vec{AB}$ is $\frac{\vec{AB}}{|\vec{AB}|}$

$$\frac{3\hat{i}+4\hat{k}}{5} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{k}$$

iii) c) $-5\hat{j}$

7- i) b- f is many-one and onto

ii) $f: R \to R$

$f(x) = 4x$

$f(x_1) = f(x_2) \Rightarrow 4x_1 = 4x_2$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one-one

$y = f(x)$

$y = 4x$

$x = \frac{y}{4}$

$f(x) = f\left(\frac{y}{4}\right)$

$= 4 \cdot \frac{y}{4}$

$f(x) = y$

$\therefore f$ is on-to

f is a bijective function

Hence f is invertible function.

$f^{-1}(x) = \frac{x}{4}$

8 E: "number 5 appears at least once"

F: "sum of the numbers appearing is 9"

$E = \{(1,5), (2,5), (3,5), (4,5), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,5)\}$

$F = \{(3,6), (4,5), (5,4), (6,3)\}$

$E \cap F = \{(4,5), (5,4)\}$

Required Probability $= P(E/F)$

$= \dfrac{P(E \cap F)}{P(F)}$

$= \dfrac{2/36}{4/36} = \dfrac{1}{2}$

9. $A = \{1,2,3\}$

$R = \{(1,3), (1,1), (2,2), (3,3), (3,1)\}$

Reflexive, $(a,a) \in R$, for every $a \in A$

$(1,1) \in R, (2,2) \in R, (3,3) \in R$

$\therefore R$ is reflexive.

Symmetric, $(a,b) \in R \Rightarrow (b,a) \in R$,
for all $a, b \in A$

$(1,3) \in R$ and $(3,1) \in R$

R is symmetric

Transitive, $(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R$,
for all $a, b, c \in A$

$(1,3) \in R, (3,1) \in R \Rightarrow (1,1) \in R$
$(3,3) \in R, (3,1) \in R \Rightarrow (3,1) \in R$
$(1,1) \in R, (1,3) \in R \Rightarrow (1,3) \in R$
$(1,3) \in R, (3,3) \in R \Rightarrow (1,3) \in R$

R is transitive

$\therefore R$ is an equivalence relation.

ii) $[1] = \{1, 3\}$

$[2] = \{2\}$

$[3] = \{1, 3\}$

10 i) $\begin{bmatrix} x & 3 \\ -2 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix} = 2\begin{bmatrix} 1 & 0 \\ y & 3 \end{bmatrix}$

$$\begin{bmatrix} x-1 & 0 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 2y & 6 \end{bmatrix}$$

$$x-1=2 \Rightarrow x=3$$

$$2y=-4 \Rightarrow y=-2$$

ii) $AB = \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & -1 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 10 & -5 & -15 \\ -4 & 2 & 6 \\ 6 & -3 & -9 \end{bmatrix}$$

11 i) $V=x^3, \; S=6x^2$

$$\frac{dV}{dt} = 24 \, cm^3/s$$

$$3x^2\frac{dx}{dt} = 24$$

$$\frac{dx}{dt} = \frac{24}{3x^2} = \frac{8}{x^2}.$$

$$S = 6x^2$$

$$\frac{dS}{dt} = 12x \frac{dx}{dt}$$

$$= 12x \cdot \frac{8}{x^2}$$

$$= \frac{12 \times 8}{x}$$

when $x = 6\,cm$

$$\frac{dS}{dt} = \frac{12 \times 8}{6}$$

$$= 16\ cm^2/s$$

ii) Minimum value of $|x| = 0$

$\therefore$ Minimum value of $|x| - 2 = 0 - 2$

$= -2$

OR

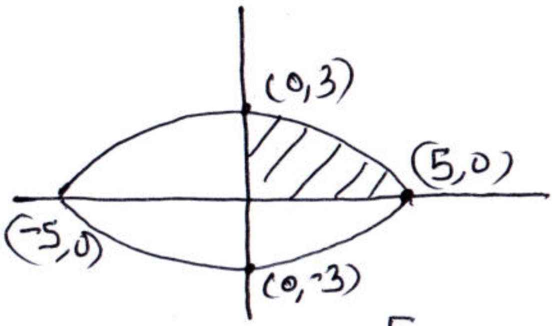
$$|x| \geq 0$$

$$|x| - 2 \geq 0 - 2$$

$$f(x) \geq -2$$

local minimum value of $f(x)$ is -2.

12. $\dfrac{x^2}{25}+\dfrac{y^2}{9}=1$



$\dfrac{x^2}{25}+\dfrac{y^2}{9}=1$

$\dfrac{y^2}{9}=1-\dfrac{x^2}{25}$

$=\dfrac{25-x^2}{25}$

$y^2=\dfrac{9}{25}(25-x^2)$

$y=\dfrac{3}{5}\sqrt{25-x^2}$.

Required Area $=4\displaystyle\int_0^5 y\,dx$

$=4\displaystyle\int_0^5 \frac{3}{5}\sqrt{25-x^2}\,dx$

$=4\times\dfrac{3}{5}\left[\dfrac{x}{2}\sqrt{25-x^2}+\dfrac{25}{2}\sin^{-1}\dfrac{x}{5}\right]_0^5$

$=4\times\dfrac{3}{5}\left[\dfrac{25}{2}\sin^{-1}1-0\right]$

$=4\times\dfrac{3}{5}\times\dfrac{25}{2}\times\pi/2$

$=15\pi$ sq. unit

13. i) a) $y = e^x + 1$

ii $\dfrac{dy}{dx} = \dfrac{\sqrt{9-y^2}}{x}$

$$\frac{dy}{\sqrt{9-y^2}} = \frac{dx}{x}$$

$$\int \frac{dy}{\sqrt{9-y^2}} = \int \frac{dx}{x}$$

$$\sin^{-1}\left(\frac{y}{3}\right) = \log|x| + C$$

14. i) $\vec{a} = 2\hat{i} + \hat{j}$, $\vec{b} = 2\vec{a}$

$$= 4\hat{i} + 2\hat{j}$$

$$\vec{a}\cdot\vec{b} = (2\hat{i}+\hat{j})\cdot(4\hat{i}+2\hat{j})$$

$$= 8 + 2 = 10$$

$$|\vec{b}| = \sqrt{4^2+2^2} = \sqrt{20}$$

Projection $\vec{a}$ on $\vec{b}$ is $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \dfrac{10}{\sqrt{20}}$

$$= \sqrt{5}$$

ii) $\vec{a} = 3\hat{i}+\hat{j}+4\hat{k}$

$\vec{b} = \hat{i}-\hat{j}+\hat{k}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= 5\hat{i}+\hat{j}-4\hat{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{25+1+16} = \sqrt{42}$$

Required area is $\sqrt{42}$ sq.unit

15. $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z}{2}$, $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+1}{2}$

$x_1=1$ $y_1=1$ $z_1=0$ $x_2=2$ $y_2=1$ $z_2=-1$

$a_1=2$ $b_1=-1$ $c_1=2$ $a_2=3$ $b_2=-5$ $c_2=2$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & -1 \\ 2 & -1 & 2 \\ 3 & -5 & 2 \end{vmatrix}$$

$$=1(-2+10)-0-1(-10+3)$$

$$= 8+7$$

$$=15$$

$$\sqrt{(a_1b_2-a_2b_1)^2+(b_1c_2-b_2c_1)^2+(a_1c_2-a_2c_1)^2}=$$

$$\sqrt{(-10-{-3})^2+(-2-{-10})^2+(4-6)^2}$$

$$=\sqrt{49+64+4}$$

$$=\sqrt{117}$$

$$S.D=\frac{15}{\sqrt{117}}$$

16. i) $P(A)=0.4 \quad P(B)=0.5$

$P(A\cap B)=0.4\times0.5 \ =0.2$

$P(\text{neither } A \text{ nor } B)=P(A\cup B)'$

$=1-P(A\cup B)$

$=1-[P(A)+P(B)-P(A\cap B)]$

$=1-[0.4+0.5-0.2]$

$=1-0.7$

$=0.3$

ii) E_1 : Getting '5' $\Rightarrow P(E_1) = 1/6$

E_2 : Not getting '5' $\Rightarrow P(E_2) = 5/6$

A : Reports '5'

$P(A/E_1) = 4/5$

$P(A/E_2) = \frac{1}{5}$

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1)\, P(A/E_1) + P(E_2)\, P(A/E_2)}$$

$$= \frac{\frac{1}{6} \times \frac{4}{5}}{\frac{1}{6} \times \frac{4}{5} + \frac{5}{6} \cdot \frac{1}{5}}$$

$$= \frac{4}{4+5}$$

$$= \frac{4}{9}$$

17. $x+2y+z=18$

$2x+y+z=5$

$x-3y+4z=3$

$$A=\begin{bmatrix}1 & 2 & 1\\ 2 & 1 & 1\\ 1 & -3 & 4\end{bmatrix},\ B=\begin{bmatrix}18\\5\\3\end{bmatrix},\ X=\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$X=A^{-1}\cdot B$$

$$|A|=\begin{vmatrix}1 & 2 & 1\\ 2 & 1 & 1\\ 1 & -3 & 4\end{vmatrix}=-14$$

$$\text{adj}\,A=\begin{bmatrix}7 & -11 & 1\\ -7 & 3 & 1\\ -7 & 5 & -3\end{bmatrix},\ A^{-1}=\frac{1}{-14}\begin{bmatrix}7 & -11 & 1\\ -7 & 3 & 1\\ -7 & 5 & -3\end{bmatrix}$$

$$X=A^{-1}\cdot B=\frac{1}{-14}\begin{bmatrix}7 & -11 & 1\\ -7 & 3 & 1\\ -7 & 5 & -3\end{bmatrix}\begin{bmatrix}18\\5\\3\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{-14}\begin{bmatrix}74\\-108\\-110\end{bmatrix}=\begin{bmatrix}\frac{-74}{14}\\ \frac{108}{14}\\ \frac{110}{14}\end{bmatrix}$$

$$x=\frac{-74}{14}=-\frac{37}{7}\qquad y=\frac{108}{14}=\frac{54}{7}\qquad z=\frac{110}{14}=\frac{55}{7}$$

18. i) $y=\sqrt{\sin x}$

$$\frac{dy}{dx}=\frac{1}{2\sqrt{\sin x}}\cdot\cos x$$

ii) $x=a(t-\sin t)$, $y=a(1+\cos t)$

$$\frac{dx}{dt}=a(1-\cos t)\ ,\quad \frac{dy}{dt}=-a\sin t$$

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{-a\sin t}{a(1-\cos t)}$$

$$=\frac{-2\sin t/2\cos t/2}{2\sin^2 t/2}$$

$$=-\cot\frac{t}{2}$$

iii) $y=x^x$

$$\log y=\log x^x$$

$$=x\log x$$

diff both sides w.r.t x

$$\frac{1}{y}\frac{dy}{dx}=x\frac{1}{x}+\log x\times 1$$

$$\frac{dy}{dx}=y(1+\log x)=x^x(1+\log x)$$

19. i) $I = \int \sin x \, e^x \, dx$

$= \sin x \int e^x dx - \int \left(\frac{d}{dx} \sin x \int e^x dx \right) dx$

$= \sin x \, e^x - \int \cos x \, e^x \, dx$

$= \sin x e^x - \left[\cos x \int e^x - \int \frac{d}{dx} \cos x \int e^x dx \right]$

$= \sin x e^x - \left[\cos x e^x - \int -\sin x e^x dx \right]$

$I = \sin x e^x - \cos x e^x - \int \sin x e^x dx$

$= e^x (\sin x - \cos x) - I$

$2I = e^x (\sin x - \cos x) + C$

$I = \frac{e^x}{2} (\sin x - \cos x) + C$

ii $\quad I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx \quad -(1)$

$$= \int_0^{\pi/2} \frac{\sin^3(\pi/2 - x)}{\sin^3(\pi/2 - x) + \cos^3(\pi/2 - x)} dx$$

$$I = \int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x + \sin^3 x} dx \quad -②$$

① + ②

$$2I = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x} dx$$

$$2I = \int_0^{\pi/2} 1 \, dx$$

$$2I = \pi/2$$

$$I = \frac{\pi}{4}$$

20. $Z = 3x + 7y$

subject to

$$3x + 4y \leq 60$$

$$x + y \geq 10$$

$$x - 3y \leq -6$$

$$x \geq 0$$

$3x+4y=60$

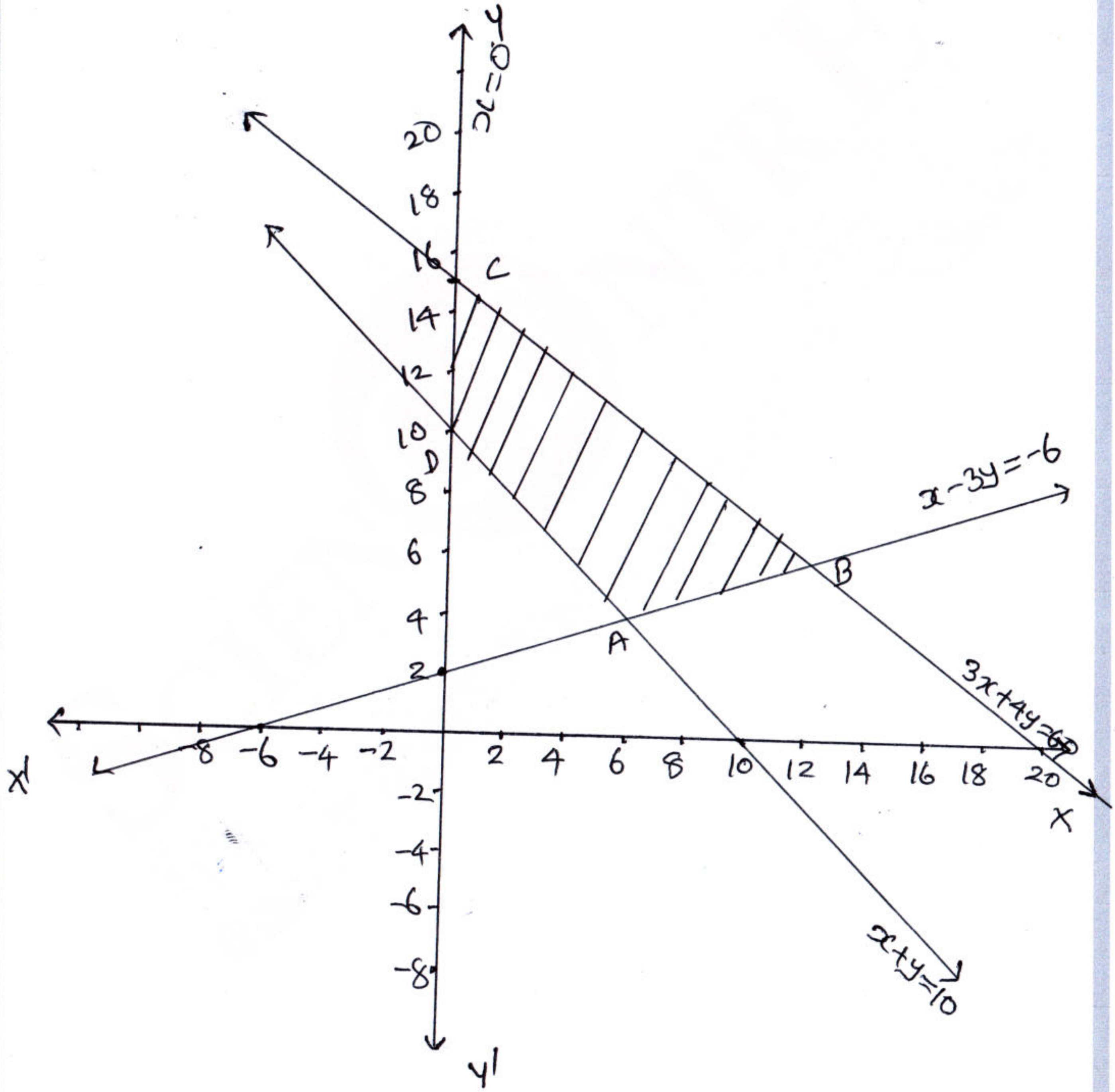
x	y
0	15
20	0

$x+y=10$

x	y
0	10
10	0

$x-3y=-6$

x	y
0	2
-6	0



Corner Point	$z = 3x + 7y$
A (6, 4)	$z = 46$
B (12, 6)	$z = 78$
C (0, 15)	$z = 105$
D (0, 10)	$z = 70$

Maximum value of z is 105 at (0, 15)